AN INTUITIONISTIC THEORY OF LAWLIKE, CHOICE AND LAWLESS SEQUENCES

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Dedicated to Stephen Cole Kleene

Abstract

In [12] we defined an extensional notion of relative lawlessness and gave a classical model for a theory of lawlike, arbitrary choice, and lawless sequences. Here we introduce a corresponding intuitionistic theory and give a realizability interpretation for it. Like the earlier classical model, this realizability model depends on the (classically consistent) set theoretic assumption that a particular Δ^2_1 well ordered subclass of Baire space is countable.

§1. Introduction.

1.1. Background. Infinitely proceeding sequences of natural numbers are the fundamental objects of L. E. J. Brouwer's intuitionistic theory of the continuum. Choice sequences are generated by more or less freely choosing one integer after another; at each stage, the chooser *may* also specify restrictions on future choices (compatible with previous restrictions, if any, and with the indefinite continuation of the process).

Brouwer called "lawlike" or "a sharp arrow" any sequence *all* of whose values are completely determined (restricted) according to some fixed law at some finite stage in the generation of the sequence. G. Kreisel [9] called "lawless" any sequence for which (i) "the *simplest kind of restriction on restrictions is made*, namely some finite initial segment of values is prescribed, and beyond this, no restriction is to be made." Kreisel and A. S. Troelstra developed a theory of lawlike and *intensionally* lawless sequences, based on (i), for which they were able to prove that every formula without free lawless variables is equivalent to one without any lawless variables and hence "it is possible to regard lawless sequences as a 'figure of speech'."[2]

Alternatively a sequence could be called lawless if (ii) it successfully evades description by any fixed law. The assumption that lawless sequences are real

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[2][15, p. 639]. Kreisel [9, p. 225] asserts however that the equivalence result is *not* to be interpreted in this way, but rather as "a complete analysis of all known properties of lawless sequences in the given context."