TEMPORAL EXPRESSIVE COMPLETENESS IN THE PRESENCE OF GAPS

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Abstract

It is known that the temporal connectives *until* and *since* are expressively complete for Dedekind complete flows of time but that the Stavi connectives are needed to achieve expressive completeness for general linear time which may have "gaps" in it. We present a full proof of this result.

We introduce some new unary connectives which, along with *until* and *since* are expressively complete for general linear time. We axiomatize the new connectives over general linear time, define a notion of complexity on gaps and show that *since* and *until* are themselves expressively complete for flows of time with only isolated gaps. We also introduce new unary connectives which are less expressive than the Stavi connectives but are, nevertheless, expressively complete for flows of time whose gaps are of only certain restricted types. In this connection we briefly discuss scattered flows of time.

§1. Introduction: the problem of expressive completeness.

This section will present the problem of expressive completeness of temporal connectives within the more general model theoretic concept of the existence of a finite G-basis for m-adic theories. The known results in this area will then be outlined.

We begin with the ordinary propositional temporal logic. Assume we are given a flow of time $(T, <)$, where T is the set of moments of time and $<$ is a transitive and irreflexive relation on T, thought of as the earlier-later relation. We define the notion of m-dimensional temporal logic on $(T, <)$. An m-dimensional atomic proposition q on $(T, <)$ can be associated with a subset Q of T^m, representing the set of all m-tuples of moments of time where q is true. The boolean logical operations on temporal formulas, such as $\wedge$, $\vee$, $\sim$ and $\rightarrow$ correspond naturally to operations on these subsets. It is clear that a temporal assignment h to the atoms associating with atoms q_i subsets $h(q_i) \subseteq T^m$, gives rise to an ordinary model for $(T, <, Q_i, =)$. To be able to express formally the connections between propositional temporal formulas and subsets of T^m, we need to use the m-adic language with $(T, <, Q_i, =)$, where $Q_i \subseteq T^m$ are m-place predicates and $=$ is equality.

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