LABELLED DEDUCTIVE SYSTEMS: A POSITION PAPER

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§1. Labelled deductive systems in context.

The purpose of this paper is to introduce a general notion of a logical system, namely that of a *Labelled Deductive System* (*LDS*), and show that many logical systems, new and old, monotonic and non-monotonic, all fall within this new framework. This research will eventually be published as a book, and this paper is based on Chapter 1 of [19].

We begin with the traditional view of what is a logical system.

Traditionally, to present a logic **L**, we need to present first the set of well-formed formulas of that logic. This is the *language* of the logic. We specify the sets of atomic formulas, connectives, quantifiers and the set of well-formed formulas. Secondly, we mathematically define the notion of consequence, that is, for sets of formulas Δ and formulas Q, we define the consequence relation $\Delta \vdash_{\mathbf{L}} Q$, which is read "$Q$ follows from Δ in the logic **L**".

The consequence relation is required to satisfy the following intuitive properties: (Δ, Δ' abbreviates $\Delta \cup \Delta'$).

Reflexivity

$$\Delta \vdash Q \text{ if } Q \in \Delta$$

Monotonicity

$$\frac{\Delta \vdash Q}{\Delta, \Delta' \vdash Q}$$

1.1. *Transitivity (cut)*

$$\frac{\Delta \vdash A; \Delta, A \vdash Q}{\Delta \vdash Q}$$

If you think of Δ as a database and Q as a query, then reflexivity means that the answer "yes" is given for any Q which is already listed in the database Δ. Monotonicity reflects the accumulation of data, and transitivity is nothing but lemma generation, namely, if $\Delta \vdash A$, then A can be used as a lemma to derive B from Δ.

These three properties have appeared to constitute minimal and most natural for a logical system, given that the main applications of logic were in mathematics and philosophy.