§11. The Construction

At last we are in a position to construct our extender sequence $\vec{E}$. We will construct the sequence $\vec{E}$ inside of V_θ where θ is least such that $L(V_\theta)$ satisfies that θ is Woodin. Note that every bounded subset of θ in $L(\vec{E})$ is in $L_\theta[\vec{E}]$ since θ is inaccessible.

The construction of $\vec{E}$ will differ from that for sequences of measures in that we do not simply define E_α by induction on α. The reason is that we want the construction to provide each E_α with an ancestry tracing back (by inverting certain collapses) to an extender on V having a certain amount of strength. The illustrious ancestry of the extenders which lie on $\vec{E}$ guarantees that all levels of $L[\vec{E}]$ are ω-iterable.

Let us call a premouse $\mathcal{M}$ *reliable* iff for all $k \leq \omega$, $\mathfrak{C}_k(\mathcal{M})$ exists and is k-iterable. We shall simply assume in this section that the premice we produce in our construction are reliable, and discharge our obligation to show this in §12.

We now define by induction on ξ a reliable coremouse $\mathcal{M}_\xi$. Simultaneously, we verify an induction hypothesis A_ξ describing the agreement between $\mathcal{M}_\xi$ and the $\mathcal{M}_\alpha$ for $\alpha < \xi$:

(A_ξ) $\mathcal{J}_\eta^{\mathcal{M}_\alpha} = \mathcal{J}_\eta^{\mathcal{M}_\xi}$ for all $\alpha < \xi$ and $\kappa \leq \inf\{\,\rho_\omega(\mathcal{M}_\nu) : \alpha < \nu \leq \xi\,\}$, where $\eta = (\kappa^+)^{\mathcal{M}_\alpha}$.

In the formulation of A_ξ, we understand that $\omega\eta = \mathrm{OR}^{\mathcal{M}_\alpha}$ in the case that $\mathcal{M}_\alpha \models \kappa^+$ doesn't exist.

We begin by setting $\mathcal{M}_0 = (V_\omega, \in, \varnothing)$. Now suppose that $\mathcal{M}_\xi$ is given and that A_ξ holds. We define $\mathcal{M}_{\xi+1}$ and verify $A_{\xi+1}$.

Case 1. $\mathcal{M}_\xi = (J_\alpha^{\vec{E}}, \varepsilon, \vec{E})$ is a passive premouse, and there are an extender F^* over V, an extender F over $\mathcal{M}_\xi$, and an ordinal $\nu < \alpha$ such that

$$V_{\nu+\omega} \subseteq \mathrm{Ult}(V, F^*)$$

and

$$F \restriction \nu = F^* \cap ([\nu]^{<\omega} \times J_\alpha^{\vec{E}})$$

and

$$\mathcal{N}_{\xi+1} = (J_\alpha^{\vec{E}}, \in, \vec{E}, \tilde{F})$$

is a 1-small, reliable premouse, with $\nu = \nu^{\mathcal{N}_{\xi+1}}$.

In this case we choose F^*, F, ν, and $\mathcal{N}_{\xi+1}$ as above with ν, the natural length of F, minimal among all such F^*. Let

$$\mathcal{M}_{\xi+1} = \mathfrak{C}_\omega(\mathcal{N}_{\xi+1}).$$

Case 2. Otherwise.