§7. THE COMPARISON PROCESS

We prove in this section a comparison lemma for 1-small mice. Our interest is not so much in the lemma itself, but in the method by which it is proved. We shall use that method in a much more important way in the next section.

For bookkeeping purposes we shall use "padded iteration trees". These are just like ordinary iteration trees except that we modify the successor clause in the definition of "iteration tree" so as to allow $\alpha T(\alpha+1)$, $\mathcal{M}_\alpha = \mathcal{M}_{\alpha+1}$, and $i_{\alpha,\alpha+1} =$ identity, and then require that $\alpha T\beta \Rightarrow \beta = \alpha+1$ or $(\alpha+1)\ T\beta$. So a padded tree is essentially an ordinary tree with the indexing of the models slowed down by repetition. We shall no doubt often fail to distinguish between iteration trees and their padded counterparts.

Theorem 7.1 (The comparison lemma). *Let $\mathcal{M}$ and $\mathcal{N}$ be n-sound, 1-small, n-iterable premice, where $n \leq \omega$. Then there are n-maximal padded iteration trees $\mathcal{T}$ on $\mathcal{M}$ and $\mathcal{U}$ on $\mathcal{N}$ such that either*

(1) $\mathcal{T}$ and $\mathcal{U}$ have successor length $\theta+1$, and either

(a) $\mathcal{M}_\theta$ is an initial segment of $\mathcal{N}_\theta$ and $D^{\mathcal{T}} \cap [0,\theta]_T = \varnothing$ and deg$(\alpha+1) = n$ for all $\alpha+1 \in [0,\theta]_T$, or

(b) $\mathcal{N}_\theta$ is an initial segment of $\mathcal{M}_\theta$ and $D^{\mathcal{U}} \cap [0,\theta]_U = \varnothing$ and deg$(\alpha+1) = n$ for all $\alpha+1 \in [0,\theta]_U$,

or

(2) $\mathcal{T}$ and $\mathcal{U}$ have limit length, one of the two is not simple, and in some $V^{\mathrm{Col}(\kappa,\omega)}$ there are wellfounded cofinal branches b of $\mathcal{T}$ and c of $\mathcal{U}$ such that either

(a) $\mathcal{M}_b$ is an initial segment of $\mathcal{N}_c$, $D^{\mathcal{T}} \cap b = \varnothing$, and deg$(\alpha+1) = n$ for all $\alpha+1 \in b$, or

(b) $\mathcal{N}_c$ is an initial segment of $\mathcal{M}_b$, $D^{\mathcal{U}} \cap c = \varnothing$, and deg$(\alpha+1) = n$ for all $\alpha+1 \in c$.

PROOF. We define by induction on γ

$$\mathcal{T} \restriction \gamma = \langle T \cap (\gamma\times\gamma), D^{\mathcal{T}} \cap \gamma, \deg^{\mathcal{T}} \restriction \gamma, \langle E^{\mathcal{T}}_\alpha, \mathcal{M}^*_{\alpha+1} \mid \alpha+1 < \gamma\rangle\rangle$$

and

$$\mathcal{U} \restriction \gamma = \langle U \cap (\gamma\times\gamma), D^{\mathcal{U}} \cap \gamma, \deg^{\mathcal{U}} \restriction \gamma, \langle E^{\mathcal{U}}_\alpha, \mathcal{N}^*_{\alpha+1} \mid \alpha+1 < \gamma\rangle\rangle$$

together with the associated $\mathcal{M}_\alpha$, $\mathcal{N}_\alpha$ for $\alpha < \gamma$, $\rho^{\mathcal{T}}_\alpha$ and $\rho^{\mathcal{U}}_\alpha$ for $\alpha+1 < \gamma$, and embeddings $i^{\mathcal{T}}_{\alpha\beta}$, $i^{\mathcal{U}}_{\alpha\beta}$ (for (α,β) as appropriate). The method for defining $\mathcal{T}$ and $\mathcal{U}$ is the standard one of "iterating the least disagreement".