Preface

The model theory of fields is a fascinating subject stretching from Tarski's work on the decidability of the theories of the real and complex fields to Hrushovksi's recent proof of the Mordell-Lang conjecture for function fields. Our goal in this volume is to give an introduction to this fascinating area concentrating on connections to stability theory.

The first paper *Introduction to the model theory of fields* begins by introducing the method of quantifier elimination and applying it to study the definable sets in algebraically closed fields and real closed fields. These first sections are aimed for beginning logic students and can easily be incorporated into a first graduate course in logic. They can also be easily read by mathematicians from other areas. Algebraically closed fields are an important examples of ω-stable theories. Indeed in section 5 we prove Macintyre's result that that any infinite ω-stable field is algebraically closed. The last section surveys some results on algebraically closed fields motivated by Zilber's conjecture on the nature of strongly minimal sets. These notes were originally prepared for a two week series of lecture scheduled to be given in Beijing in 1989. Because of the Tiananmen square massacre these lectures were never given.

The second paper *Model theory of differential fields* is based on a course given at the University of Illinois at Chicago in 1991. Differentially closed fields provide a fascinating example for many model theoretic phenomena (Sacks referred to differentially closed fields as the "least misleading example"). This paper begins with an introduction to the necessary differential algebra and elementary model theory of differential fields. Next we examine types, ranks and prime models, proving among other things that differential closures are not minimal and that for κ > ℵ₀ there are 2κ non-isomorphic models. We conclude with a brief survey of differential Galois theory including Poizat's model theoretic proof of Kolchin's result that the differential Galois group of a strongly normal extension is an algebraic group over the constants and the Pillay-Sokolovic result that any superstable differential field has no proper strongly normal expansions. Most of this article can be read by a beginning graduate student in model theory. At some points a deeper knowledge of stability theory or algebraic geometry will be helpful.

When this course was given in 1991 there was an annoying gap in our knowledge about the model theory of differentially closed fields. Shelah had proved Vaught's conjecture for ω-stable theories. Thus we knew that there were either ℵ₀ or 2ℵ₀ non-isomorphic countable differentially closed fields, but did not know which. In 1993 Hrushovski and Sokolovic showed there are 2ℵ₀. The proof used the Hrushovski-Zilber work on Zariski geometries and Buium's work on abelian varieties and differential algebraic groups. This circle of ideas is also crucial to Hrushovski's proof of the Mordell-Lang conjecture for function fields. The third paper, *Differential algebraic groups and the number of countable...*