# Replacement $\not\leftrightarrow$ Collection *

Andrzej M. Zarach

Department of Mathematics, East Stroudsburg University
East Stroudsburg, PA 18301, U.S.A.
E-mail: amzarach@esu.edu

**Summary.** Let M be a transitive model for ZF(or [see D] Basic Set Theory + the Collection Scheme). Let $\langle P_i : i \in I \rangle \in$M be a family of notions of forcing and Q be the weak product of $\omega$ copies of the family. If H is Q-generic over M and N $= \bigcup\{$M[H$|s$]: $s \subset I \times \omega$, $s$-finite$\}$, then always N$\models$BST + the Replacement Scheme but not necessary N$\models$the Collection Scheme. If M$\models$ZF, then the Power Set Axiom does not hold in N but N$\models \forall\alpha\exists\beta(\beta = \aleph_\alpha)$. If M$\models$WOP (the Well-Ordering Principle), then N$\models$WOP. Thus, even in Set Theory with WOP and as many alephs as ordinals, the principle of collecting sets in a definable manner does not support the principle of collecting sets in a loose manner.

## 1. Reconstruction.

I have been inspired by Michael Hallett's "Cantorian Set Theory and Limitation of Size," [see H], to look for justifiable Cantorian set theories different than ZF or ZFC but equiconsistent with ZF.

According to [3], p.73, Cantor claimed in his letter to Mittag-Leffler of 14 November 1884 that the continuum could not be of the second power, even more that '...it has no power specifiable by a number,'(Cantor changed his mind next day). In 1904, Jules König from Budapest presented a paper at the Third International Congress of Mathematicians which claimed that the power of Cantor's continuum was not an aleph at all.

Cantor stated a few times that by sets he meant to include only well-orderable collections that could be joined by some rule into a whole ([3], p.245). He clearly believed in existence of $\aleph_\alpha$ for every ordinal $\alpha$.

If every set is well-orderable and the power of the continuum is not an aleph, then the Power Set Axiom cannot be accepted, so we need another principle that generates alephs. It is the Hartog's functional.

Let $\aleph(x) = \{\alpha : \exists f(f : \alpha \xrightarrow{1-1} x)\}$. Then ZFH! = BST + the Replacement Scheme + $\forall x \exists \beta(\aleph(x) \subseteq \beta)$ and ZFH = ZFH! + the Collection Scheme are theories pretty close to ZF that do not exclude WOP and the continuum being non-well-orderable by any class.

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