A Uniform Theorem Proving Tableau Method for Modal Logic*

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Summary. In this paper, we propose a uniform theorem proving tableau method for a wide class of systems in propositional modal logic. The class is wide enough to include most well-known systems. In this method, for a given natural number μ, a modal formula θ is effectively transformed to a first-order formula $\Delta(\theta)_\mu$ without depending on the system addressed. The transformation is based on the idea of tableau methods. Now, if S is a system that is complete for a class of Kripke frames characterized by a first-order formula Σ, then $S \vdash \theta$ iff $\Sigma \supset \Delta(\theta)_\mu$ is provable in first-order logic for some μ. This method also raises questions that are interesting from a theoretical viewpoint.

1. Introduction

In this paper, we propose a uniform tableau method for a wide class of systems in propositional modal logic. Tableau methods are efficient ways of theorem proving, based on the idea of model elimination [4], [6]. These methods are known to be especially useful for modal logic. Tableau methods have been proposed for a number of well-known systems: K, D, T, B, S4, and S5. However, there are many systems to which tableau methods have not been proposed yet. For example, S4.1 and S4.2 are not covered. Moreover, each of the proposed methods requires a system dependent individual device to achieve a complete theorem proving procedure. At present, there is no general strategy for obtaining tableau methods for all systems.

The aim of this paper is to propose a uniform tableau method applicable to a wide class of systems. The class consists of complete normal systems which are complete for a class of Kripke frames characterized by a first-order formula. The class is natural and wide enough to include most well-known systems. In our method, a given modal formula θ is effectively transformed to a first-order formula $\Delta(\theta)_\mu$ for a given natural number μ. This transformation is independent of the system being addressed. We have the following correctness theorem.

If S is a system, complete for a class of Kripke frames characterized by a first-order formula Σ, then $S \vdash \theta$ iff $\Sigma \supset \Delta(\theta)_\mu$ is provable in first-order logic for some μ.

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* This paper is in its final form and no similar paper has been or is being submitted elsewhere.