K-graph Machines: generalizing Turing's machines and arguments *

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Summary. The notion of *mechanical process* has played a crucial role in mathematical logic since the early thirties; it has become central in computer science, artificial intelligence, and cognitive psychology. But the discussion of Church's Thesis, which identifies the informal concept with a mathematically precise one, has hardly progressed beyond the pioneering work of Church, Gödel, Post, and Turing. Turing addressed directly the question: *What are the possible mechanical processes a human computor can carry out in calculating values of a number-theoretic function?* He claimed that all such processes can be simulated by machines, in modern terms, by deterministic Turing machines. Turing's considerations for this claim involved, first, a formulation of boundedness and locality conditions (for linear symbolic configurations and mechanical operations); second, a proof that computational processes (satisfying these conditions) can be carried out by Turing machines; third, the central thesis that all mechanical processes carried out by human computors must satisfy the conditions. In Turing's presentation these three aspects are intertwined and important steps in the proof are only hinted at. We introduce *K-graph machines* and use them to give a detailed mathematical explication of the first two aspects of Turing's considerations for general configurations, i.e. K-graphs. This generalization of machines and theorems provides, in our view, a significant strengthening of Turing's argument for his central thesis.

Introduction

Turing's analysis of effective calculability is a paradigm of a foundational study that (i) led from an informally understood concept to a mathematically precise notion, (ii) offered a detailed investigation of the new mathematical notion, and (iii) settled an important open question, namely the *Entscheidungsproblem*. The special character of Turing's analysis was recognized immediately by Church in his review of Turing's 1936 paper. The review was published in the first issue of the 1937 volume of the Journal of Symbolic Logic, and Church contrasted in it Turing's mathematical notion for effective calculability (via idealized machines) with his own (via λ-definability) and Gödel's general recursiveness and asserted: "Of these, the first has the advantage of making the identification with effectiveness in the ordinary (not explicitly defined) sense evident immediately...."

Gödel had noticed in his (1936) an "absoluteness" of the concept of computability, but found only Turing's analysis convincing; he claimed that Turing's work provides "a precise and unquestionably adequate definition of the

* This paper is in its final form and no similar paper has been or is being submitted elsewhere.