Contributions of K. Gödel to Relativity and Cosmology *

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Summary. K Gödel published two seminal papers on general relativity theory and its application to the study of cosmology. The first examined a non-expanding but rotating solution of the Einstein field equations, in which causality is violated; this lead to an in-depth examination of the concepts of causality and time in curved space-times. The second examined properties of a family of rotating and expanding spatially homogeneous solutions of the Einstein equations, which was a forerunner of many studies of such cosmologies. Together they stimulated examination of themes that were fundamental in the development of the Hawking-Penrose singularity theorems and in studies of cosmological dynamics. I review these two papers, and the developments that resulted from them.

1. Introduction

Gödel became interested in general relativity theory while he and Einstein were both on staff of the Institute for Advanced Studies in Princeton. Apparently they discussed the subject together often. His resultant two papers had a major impact:

> Curiously, the beginning of the modern studies of singularities in general relativity in many ways had its seeds in the presentation by Kurt Gödel (1949) of an exact solution of Einstein's equations for pressure-free matter, which could be thought of as a singularity-free, rotating but non-expanding cosmological model ... [this paper] was one of the papers presented in a special issue of *Reviews of Modern Physics* dedicated to Einstein on his 70th birthday. Gödel used this space-time as an example helping to clarify the nature of time in general relativity, for it is an exact solution of the Einstein equations in which there are closed timelike lines: an observer can travel into his own past, and (as an old man) stand alongside himself (as a young man). He shortly thereafter published a further paper (1952) discussing a family of exact solutions of Einstein's equations representing rotating and expanding spatially homogeneous universe models (and relying on the geometric results derived many decades earlier by Sophus Lie and Luigi Bianchi). As these permit non-zero redshifts, they could include realistic models of the observed universe.

* This paper is in its final form and no similar paper has been or is being submitted elsewhere