

CHAPTER 5

APPLICATIONS TO PARTICULAR SENTENTIAL LOGICS

In this chapter we determine the classes of \mathcal{S} -algebras and of full models for several logics, especially for some which do not fit into the classical approaches to the algebraization of logic. We classify them according to several of the criteria we have been considering, i.e., the properties of the Leibniz, Tarski and Frege operators, which determine the classes of selfextensional logics, Fregean logics, strongly selfextensional logics, protoalgebraic logics, etc. We also study the counterexamples promised in the preceding chapters of this monograph.

It goes without saying that the number of cases we have examined is limited, and that many more are waiting to be studied³². In our view this is an interesting program, especially for non-algebraizable logics. Among those already proven in Blok and Pigozzi [1989a] not to be algebraizable we find many quasi-normal and other modal logics like Lewis' S1, S2 and S3, entailment system E, several purely implicational logics like BCI, the system R_{\rightarrow} of relevant implication, the "pure entailment" system E_{\rightarrow} , the implicative fragment $S5_{\rightarrow}$ of the Wajsberg-style version of S5, etc. Other non-algebraizable logics not treated in the present monograph are Da Costa's paraconsistent logics C_n (see Lewin, Mikenberg, and Schwarze [1991]), and the "logic of paradox" of Priest [1979] (see Pynko [1995]). This program is also interesting for some algebraizable logics whose class of \mathcal{S} -algebras is already known, but whose full models have not yet been investigated; this includes Łukasiewicz many-valued logics (see Rodríguez, Torrens, and Verdú [1990]), BCK logic and some of its neighbours (see Blok and Pigozzi [1989a] Theorem 5.10), the equivalential fragments of classical and intuitionistic logics

³²The full models of several subintuitionistic logics have been determined in Bou [2001]; those of certain positive modal logics have been studied in Jansana [2002]; those of the version of Łukasiewicz logic that preserves degrees of truth, in Font, Gil, Torrens, and Verdú [2006]; and, more in general, those of any logic preserving degrees of truth with respect to a variety of residuated lattices (see Galatos, Jipsen, Kowalski, and Ono [2007]) are determined in Bou, Esteva, Font, Gil, Godo, Torrens, and Verdú [2009]. Most of these logics are non-protoalgebraic.