CHAPTER 4

ABSTRACT LOGICS AS MODELS
OF GENTZEN SYSTEMS

In this chapter we will introduce the issue of considering abstract logics as models of Gentzen systems, and characterize a kind of sentential logics whose full models can be described as, essentially, the models of some Gentzen system. We will also relate our study with the theory of the *algebraization of Gentzen systems*; this generalization of Blok and Pigozzi's theory of the algebraization of sentential logics was begun in Rebagliato and Verdú [1993] for some particular cases, and the general theory has started to be developed in Rebagliato and Verdú [1995][26]. We will treat some general material in Section 4.1, and in Sections 4.2 and 4.3 two particular cases will be studied, where things behave quite well. As a by-product we will get interesting results about properties of sentential logics; in particular, the open problem presented in Chapter 2 (page 48) will be solved for two important classes of logics.

Note that while in the literature Gentzen systems are mostly used to reason about their derivable sequents, in principle nothing prevents us from considering the relation of derivability of a sequent from other sequents; it is in this sense that we consider Gentzen systems in this monograph, that is, as a kind of *sequential logic*, a relation of consequence operating on sequents rather than on formulas, whose axioms are called *initial sequents* and whose theorems are called *derivable sequents* in the standard terminology. As a matter of fact, many particular *Gentzen calculi* exist in the literature having some particular axioms (i.e., initial sequents) besides the sequent $\varphi \vdash \varphi$, so one can just generalize this procedure. We will use the symbol $\mathbin{\vert\!\sim}_{\mathfrak{G}}$ to denote this relation of derivability; thus when we write

$$\{\Gamma_i \vdash \varphi_i : i \in I\} \mathbin{\vert\!\sim}_{\mathfrak{G}} \Gamma \vdash \varphi$$

we mean that there is a derivation of the sequent $\Gamma \vdash \varphi$ using the rules of the

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[26]Later papers that have somehow continued the same trend are Pynko [1999] and Raftery [2006].