CHAPTER 2

ABSTRACT LOGICS AS MODELS OF SENTENTIAL LOGICS

In this chapter we consider abstract logics as models of sentential logics. Abstract logics are suitable for modelling the metalogical properties that sentential logics can have; in this they differ notably from matrices. Our purpose is to single out for any sentential logic a class of abstract logics that exhibit some crucial metalogical properties of it. This leads us to distinguish two types of models for a sentential logic, the models "tout court" and the full models. The latter will be suitable for our purpose of modelling metalogical properties, an issue that will be dealt with specifically in the last section of this chapter, and also in Chapter 4.

2.1. Models and full models

We begin by using an abstract logic to define a logic on the algebra of formulas by the ordinary semantic procedure; using it the notion of model will be introduced.

DEFINITION 2.1. *If $\mathbb{L} = \langle \boldsymbol{A}, \mathrm{C} \rangle$ is any abstract logic, the relation $\models_{\mathbb{L}}$ induced by $\mathbb{L}$ on the formula algebra is defined, for any $\mathit{\Gamma} \cup \{\varphi\} \subseteq Fm$, by:*

$$\mathit{\Gamma} \models_{\mathbb{L}} \varphi \iff \textit{for any } h \in \mathrm{Hom}(\boldsymbol{Fm}, \boldsymbol{A})\,,\ h(\varphi) \in \mathrm{C}\big(h[\mathit{\Gamma}]\big)\,.$$

If L is any class of abstract logics, then it induces on the formula algebra the relation $\models_{\mathsf{L}} \ = \bigcap\{\models_{\mathbb{L}} : \mathbb{L} \in \mathsf{L}\}$.

PROPOSITION 2.2. *The relations $\models_{\mathbb{L}}$ and $\models_{\mathsf{L}}$ defined on the formula algebra $\boldsymbol{Fm}$ are structural consequence relations on this algebra.*

PROOF. It is easy to see that $\models_{\mathbb{L}}$ is a consequence relation, that is, that the operator defined as $\varphi \in \mathrm{Cn}_{\mathbb{L}}(\mathit{\Gamma})$ iff $\mathit{\Gamma} \models_{\mathbb{L}} \varphi$ is a closure operator on $\boldsymbol{Fm}$. Actually, $\mathrm{Cn}_{\mathbb{L}}$ is the abstract logic on $\boldsymbol{Fm}$ projectively generated from $\mathbb{L}$ by the family of all homomorphisms $\mathrm{Hom}(\boldsymbol{Fm}, \boldsymbol{A})$. By Theorem XII.2 of Brown and