

INTRODUCTION

The purpose of this monograph is to develop a very general approach to the algebraization of sentential logics, to show its results on a number of particular logics, and to relate it to other existing approaches, namely to those based on logical matrices and the equational consequence developed by Blok, Czelakowski, Pigozzi and others.

The main distinctive feature of our approach lies in the mathematical objects used as models of a sentential logic: We use *abstract logics*¹, while the classical approaches use *logical matrices*. Using models with more structure allows us to reflect in them the metalogical properties of the sentential logic. Since an abstract logic can be viewed as a “bundle” or family of matrices, one might think that the new models are essentially equivalent to the old ones; but we believe, after an overall appreciation of the work done in this area, that it is precisely the treatment of an abstract logic as a single object what gives rise to a useful—and beautiful—mathematical theory, able to explain the connections, not only at the logical level but at the metalogical level, between a sentential logic and the particular class of models we associate with it, namely the class of its *full models*.

Traditionally logical matrices have been regarded as the most suitable notion of model in the algebraic studies of sentential logics; and indeed this notion gives several completeness theorems and has generated an interesting mathematical theory. However, it was not clear how to use the matrices in order to associate a class of algebras with an arbitrary sentential logic, in a general way that could be mathematically exploited in order to find and study the connections between the properties of the sentential logic and the properties of the class of algebras; and this was true in spite of the fact that in most of the best-known logics these connections were recognized early. Rasiowa singled out in her [1974] the *standard systems of implicative extensional propositional calculi*, based on an implication

¹In our own later publications we have preferred the term *generalized matrices* over that of *abstract logics*, in order to avoid any misunderstanding with concepts in abstract model theory. See Font [2003b] and Font, Jansana, and Pigozzi [2001], [2003], [2006].