6. Canonization Problems

This is the first of two chapters dealing with canonization. In this chapter we consider canonization up to logical equivalences $\equiv^{\mathcal{L}}$, in particular for the logics $\mathcal{L} = L^k_{\infty\omega}$ and $C^k_{\infty\omega}$. We investigate the relation between PTIME canonization, PTIME inversion of the invariants, and the existence of recursive presentations and normal forms for related fragments of PTIME. It is shown for instance that PTIME invertibility for the I_{C^k} for all k would imply that FP+C captures exactly all queries that are PTIME computable and $C^\omega_{\infty\omega}$-definable. This and similar implications are of a hypothetical status, however: the problem of PTIME invertibility — and of PTIME canonization for C^k and L^k — remains open for arbitrary k. We show in this chapter that the general case essentially reduces to that for the *three* variable fragments. An explicit solution to the problem for the *two* variable fragments will be presented in the next chapter.

• Section 6.1 reviews the general notion of canonization and discusses canonization with respect to isomorphism in connection with algorithms on structures.

• In Section 6.2 PTIME canonization for $\equiv^{\mathcal{L}}$ is related to recursive presentations of fragments of PTIME.

• Section 6.3 discusses PTIME inversion of the I_{C^k} and I_{L^k} in relation to canonization and normal forms for the related fragments of PTIME. In particular we present theorems on the impact of PTIME invertibility of all I_{C^k}, respectively all I_{L^k} (in the sense of Definition 6.9), on the classes PTIME $\cap\, C^\omega_{\infty\omega}$ and PTIME $\cap\, L^\omega_{\infty\omega}$.

• The reduction of these results to the three variable fragments is presented in Section 6.4.

6.1 Canonization

For the general notion of canonization compare Definition 1.57 and related remarks in Section 1.7.1. Formally a function H provides canonization for $\sim$ if it satisfies two conditions. For all x we want $H(x) \sim x$ and whenever