5. Related Lindström Extensions

In this chapter FP+C is shown to be more expressive than the natural extensions of fixed-point logic by cardinality Lindström quantifiers.

- Section 5.1 introduces a structural padding technique that is suitable for the proof of this separation result. More generally, this technique serves to expose weaknesses of quantifier extensions in the case that these quantifiers do not have the right scaling properties with respect to certain extensions of structures.

- This technique is applied in Section 5.2 to show that FP($\mathcal{Q}_{\mathrm{card}}$) cannot express all FP+C-definable boolean queries. The same applies to FP($\mathcal{Q}^{\sim}_{\mathrm{card}}$) with quantifiers for all cardinality properties based on the counting of equivalence classes. In fact the separation even establishes that not all of FP* can be captured by these quantifier extensions.

- In Section 5.3 we apply the padding technique to derive corollaries concerning the weakness of two other quantifier classes. The classes of all properties of rigid structures and that of all properties of sparse structures, respectively, are shown to fall short of FP* and in particular of PTIME.

In the previous chapter FP+C has been characterized as the natural extension of fixed-point logic that incorporates expressive means for dealing with cardinalities and corresponding arithmetic. Recall that a main feature of the formalization was the introduction of a second, arithmetical sort. This type of a *functorial extension* — based partly on the manipulation of the structures under consideration — is intuitively different from the established formalism for extensions in abstract model theory, namely that of Lindström extensions or extensions through generalized quantifiers. Can this difference in appearance be substantiated in more rigorous terms? There is some sense in which this cannot be achieved: it is a known fact that the Lindström approach to extensions of logics is sufficiently general to describe any reasonable extension of first-order logic, more precisely any extension with the appropriate closure properties. No doubt therefore FP+C is equivalent with a Lindström extension of first-order logic, and also with a Lindström extension of fixed-point logic. As FP+C is a logic with recursive syntax and semantics these Lindström extensions can trivially be chosen to use recursive families