3. The Invariants

We introduce *complete structural invariants* that classify finite relational structures up to C^k- and L^k-equivalence, respectively. These invariants are based on the definable pre-orderings with respect to C^k- and L^k-types obtained in the analysis of the games in the preceding chapter. The invariants are PTIME computable and inherit specific definability properties from the pre-orderings with respect to types. These definability properties and a close relationship with the fixed-point logics make the invariants extremely useful in investigations concerning fixed-point logics and complexity issues. This approach has been initiated and lead to success in the seminal work of Abiteboul and Vianu. They first introduced a kind of ordered invariants with respect to their model of relational computation and with this technique derived important results concerning the relationship between FP and PFP.

- In the introductory Section 3.1 we relate the concept of the proposed invariants to the abstract notion of complete invariants.

- Section 3.2 provides the definition of our C^k-invariants and states their fundamental definability properties.

- Section 3.3 similarly treats the invariants for L^k.

- In Section 3.4 we consider applications of the invariants to the analysis of fixed-point logics. A main point is the discussion of the Abiteboul-Vianu Theorem on the relation between FP and PFP. As far as the C^k-invariants are concerned, the corresponding considerations are of a preliminary nature here. This analysis will be pursued further in Chapter 4 where it becomes possible to link the C^k-invariants directly with fixed-point logic with counting. We include here a comparison between the C^k- and the L^k-invariants.

- In Section 3.5 it is indicated that — up to interpretability in powers — our invariants essentially reduce to the two-dimensional ones, i.e. to those for C^2 and L^2.