1. PRELIMINARIES

In this chapter we introduce the basic notation and terminology which will be used throughout this book. We also state a number of basic Facts. These Facts will not be proved; some of them are rather obvious (and easy to believe), others are substantial and well–known theorems; further Facts will be stated when they are needed. These Facts are sufficient for most of the proofs in this book; the chief exceptions are the proofs of Theorems 3.5 and 6.4. Finally, we prove the very important fixed point lemma (Lemma 1) and apply it to prove that Robinson's Arithmetic Q is essentially undecidable (Theorem 2) and that in extensions of Q there are no truth–definitions (Theorem 3).

The language L_A of elementary arithmetic can be described as follows. The alphabet consists of:

the propositional connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$,
the quantifiers: $\exists, \forall$,
the equality symbol: $=$,
symbols used to form (individual) variables: v, $'$,
parentheses: (,),
the arithmetical constants: 0, S, +, $\times$.

(The intended interpretation of S is the successor function.) Thus, the alphabet is finite. The *variables* of L_A are the expressions v, v$'$, v$''$, etc. We write v_n for v followed by n occurrences of $'$. In most contexts x, y, z, u, v, w, possibly with subscripts etc., will be used for variables. The terms, formulas, and sentences of L_A are defined as usual. Among the terms we distinguish the *numerals* 0, S0, SS0, SSS0, ... These will be written 0, 1, 2, ... Thus, we shall omit bars and other devices ordinarily used to indicate numerals (or Gödel numbers) and use the same symbols for natural numbers and for formal numerals. In most cases this will cause no trouble as long as the symbols for formal variables are kept strictly apart from the symbols for natural numbers (and numerals). For the latter we use k, m, n, p, q, r, s, possibly with subscripts etc. and symbols for formulas (see below). N is the set of natural numbers.

For sentences and formulas of L_A we use lower case Greek letters. Sentences will be written as φ, ψ, θ, χ, etc. and formulas as $\alpha(x)$, $\beta(x,y)$, $\gamma(x)$, $\xi(x)$, $\eta(x_1,...,x_n)$, $\rho(x,x')$, $\tau(x)$, ξ, γ, etc. The variables displayed are almost always exactly the free variables of the formula. $\xi(y)$ is obtained from $\xi(x)$ by replacing x by y, assumed not to be free in $\xi(x)$, and, possibly renaming bound variables in the usual way. $\xi(k)$ is obtained from $\xi(x)$ by replacing x by the numeral k (or, if you prefer, by the numeral for the number k). This generalizes in the obvious way to substitutions involving more than one variable. We use := to denote equality between formulas.

By a *theory* T we understand a set of sentences (to be thought of as the (nonlogical) axioms of T). (It would be inconvenient to identify a theory T with the set of