Zil'ber's Trichotomy and o-minimal Structures

(Extended abstract) ⋆

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A first-order structure $\mathcal{M}$ is called *a geometric structure* (see [2]) if it has the following properties:
(i) $acl(-)$ satisfies the Exchange principle.
Namely, given $a, b, \bar{c}$ from M, if $a \in acl(b, \bar{c}) \setminus acl(\bar{c})$ then $b \in acl(a, \bar{c})$.
(ii) For any formula $\varphi(\bar{x}, \bar{y})$ there is $n \in \mathbb{N}$ such that for any $\bar{b}$ in M, $\varphi(\bar{x}, \bar{b})$ has either less than n solutions in M or infinitely many.

O-minimal and strongly minimal structures are geometric structures. The field of p-adics and pseudo-finite fields are geometric structures as well.

Given a geometric structure $\mathcal{M}$ one can assign a dimension to definable sets in a natural way which in all the field cases mentioned above is just the algebro-geometric dimension of the Zariski closure. *A curve* is any definable 1-dimensional subset of M^2 and a definable (or interpretable) family $\mathcal{F}$ of curves is called *normal* if any two curves from $\mathcal{F}$ which are given by different parameters intersect at most finitely many times. If $\mathcal{F}$ is normal its dimension is taken to be the dimension of the parameter set.

Given a geometric structure, one and only one of the following holds.

Z1. Every interpretable normal family of curves $\mathcal{F}$ is of dimension at most 1 and for all but finitely many curves $\mathcal{C} \in \mathcal{F}$, for all but finitely many points $\langle a, b \rangle \in \mathcal{C}$, either $\dim(\mathcal{C} \cap (\{a\} \times M)) = 1$ or $\dim(\mathcal{C} \cap (M \times \{b\})) = 1$.

Z2. Every interpretable normal family of curves is of dimension at most 1, but Z1 does not hold.

Z3. There is an interpretable normal family of curves of dimension greater than 1.

In the early 1980's Boris Zil'ber (see [6]), in his analysis of $\aleph_1$-categorical structures, suggested that the above trichotomy corresponds to the interpretability (or the lack of which) of certain algebraic structures in $\mathcal{M}$. He called Z2 and Z3 the module-like and field-like cases, respectively, and conjectured that if a strongly minimal structure satisfies Z3 then it can interpret a field. We formulate this correspondence as follows:

Definition 1 A class $\mathcal{K}$ of geometric structures is said to satisfy the Zil'ber Principle, ZP, if for every $\mathcal{M} \in \mathcal{K}$,

⋆ This abstract discusses a joint work of the author and Sergei Starchenko. For the proof of the main theorem see [4]. An expanded version of this abstract has appeared in [5].