Strongly Minimal Sets and Geometry

David Marker *

Department of Mathematics
University of Illinois at Chicago
851 South Morgan Street, Chicago, IL 60607-7045, USA
marker@math.uic.edu

These lectures will provide a brief introduction to the model theory of strongly minimal sets. The first two sections will develop the basic combinatorial geometry of strongly minimal sets. In sections three and four we will show how the pregeometry of the strongly minimal set detects the presence of ambient algebraic structure. Finally we will show how these ideas come together in Hrushovski's proof ([H1]) of the Mordell-Lang conjecture for function fields.

Strongly minimal sets are just the beginning of the story in *geometric model theory.* My hope is that by concentrating on strongly minimal sets I can give a reasonably self-contained introduction to this important and beautiful subject. We assume only that the reader is familiar with the treatment of ω-stable theories given in [CK] or [S]. A full development of geometric model theory is given in [P1].

My own appreciation of geometric model theory was slow in coming. I owe a great deal to Anand Pillay, Elisabeth Bouscaren and John Baldwin for the numerous conversations it took to enlighten me.

1 Strongly minimal sets and pregeometries

Let $\mathcal{L}$ be a first order language and let M be an $\mathcal{L}$-structure. Recall that a formula $\phi(\bar{v})$ with parameters from M is said to be *strongly minimal* if for any elementary extension N of M and any formula $\psi(\bar{v})$ with parameters from N exactly one of $\{\bar{a} \in N : N \models \phi(\bar{a}) \wedge \psi(\bar{a})\}$ and $\{\bar{a} \in N : N \models \phi(\bar{a}) \wedge \neg\psi(\bar{a})\}$ is infinite. We say that a subset D of M^n is strongly minimal if it is defined by a strongly minimal formula. We will often consider D as a structure in its own right by taking all of the structure induced from definable subsets in M^n (by "definable" I will always mean "definable with parameters" unless I specify otherwise).

If $A \subset M$ and $b \in M$ we say that b is *algebraic* over A if there is an $\mathcal{L}$-formula $\phi(v)$ with parameters from A such that $M \models \phi(b)$ and $\{a \in M : M \models \phi(a)\}$ is finite. The set of all elements of A algebraic over A is called the *algebraic closure* of A and denoted $\operatorname{acl}(A)$. The algebraic closure relation on a strongly minimal set determines a pregeometry.

* Partially supported by NSF grant DMS-9306159 and an American Mathematics Society Centennial Fellowship.