Towards a Categorical Foundation of Mathematics

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1. Introduction

This paper is an expanded version of a talk with the same title given at the Logic Colloquium in Haifa in August, 1995. A year later, at the time of writing these lines, one of the papers, [M2], on which the talk was based has appeared in print. The present paper is, primarily, a somewhat philosophical introduction to the research monograph [M3], the other source of the talk. For understanding the story, it is necessary to keep the contents of [M2] in view; some of [M2] will be recapitulated here. The paper is written for readers with at least a slight familiarity with Category Theory. At certain points, we make remarks referring to more advanced notions; however, the understanding of these is not essential for the main part of the paper.

Some more remarks on the background.

In the abstract for the Haifa talk, another, related development, the material of the papers [M1], was mentioned. In the talk itself, there was no time to enter the subject of sketches. Because of similar considerations of space, sketches will not be discussed in this paper either.

Higher dimensional categories played an important, albeit somewhat hypothetical, role in the talk. They were intended to form the universe of the proposed foundation of the title; on the other hand, their very definition had not yet been given in arbitrary dimensions. In the meantime, a breakthrough has taken place; John Baez and James Dolan proposed (see [BD2]), in a somewhat sketchy form, a definition for the concept of "weak n -category", for all natural numbers. I believe that the Baez/Dolan definition is indeed satisfactory. In particular, I think that their "weak n-categories" should, collectively for all n and endowed with a suitable attendant structure connecting them to each other, be adopted as forming the universe of the new foundation I am envisaging. Besides, the Baez/Dolan definition also contains very interesting conceptual innovations that make it striking and illuminating even when specialized to the otherwise well-known case of $n = 2$! On the other hand, the Baez/Dolan proposal is not the end of the work of even giving the definition of the basic concepts surrounding

* The author's research is supported by NSERC Canada and FCAR Quebec