

V=L and Maximize

Penelope Maddy

Department of Philosophy
University of California
Irvine, CA 92717, USA
pjmaddy@uci.edu

The problem that interests me is easy to state: what justifies the axioms of set theory? Some observers suggest that decisions on the adoption or rejection of axiom candidates are not made on rational grounds, that they are subject only to psychological or sociological or aesthetic constraints. This may be right, but I think it is too early to concede the point; my working hypothesis is that there are sound arguments to be made on these issues. As a student of the methodology of set theory, my job is to try to isolate and elaborate these sound arguments.

In Zermelo's day, for many of the axioms in his first list, the problem of justification was somewhat less daunting than it is for the candidates in dispute today. Zermelo was faced with a fairly well-developed body of set theoretic lore; the difficulty was that paradoxes and other uncertainties lurked around its edges. His goal was to select some particularly fundamental statements to serve as starting points; this was to be done skillfully, so that the core doctrine could be deduced without the troublesome outliers. The upshot, for many of his axioms, was that certain previously-accepted claims were being promoted to axiomatic status, not that any new claims were being made. The selection of these particular statements for promotion needed justification—presumably on grounds of economy, efficiency, and likely consistency—but the problem of justifying the statements themselves was eased by their previous acceptance.¹

The case of the axiom of choice was different. Choice was not a previously-accepted or uncontroversial claim; what needed justification was not its mere promotion, but the statement itself. With this case, the problem of what justifies an axiom arose in a more pressing and poignant form, as did the prior problem of what sorts of grounds are appropriate for such justifications. The subsequent, fascinating history is familiar, and the outcome is now stable, so I won't rehearse it here. But it is important to note that since then, with the advent of independence results and the subsequent search for new, stronger axioms, the problem of justification has become ever more acute.

Rather than talk in the abstract about the general problem of justifying axioms, I want to focus on one particularly salient case: the axiom of constructibility. The decision on $V=L$ is the first truly momentous one after ZFC, and it is pivotal for the further development of the subject. Despite the fact that adding $V=L$ is a safe, economical, and powerful option, settling many of the lingering

¹ This is not to say that all Zermelo's axioms, even excluding choice, were uncontroversial or that his system was immediately accepted. See Moore [1982], pp. 160-167, for details.