

Two-Dimensional Temporal Logic

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ABSTRACT Two-dimensional combinations of temporal and modal logics have been studied for some time for their logical properties and their applications to natural language semantics and computer science. In this survey, we briefly describe a variety of these logics, concentrating on the temporal-temporal combinations, their properties and uses. We also look at some more recent results using irreflexivity rules, tiling and mosaic techniques.

1 Introduction

We survey some recent results about various two-dimensional temporal logics and some similar modal-temporal logics. We look at their simple logical properties and applications in computer science and artificial intelligence. For a more general account of multi-dimensional modal logic see [MV97] and for broader surveys of temporal logic see [GHR95].

The logics we are most concerned with are defined over frames consisting of a cross product of simpler structures. Valuations of propositional atoms will be made at ordered pairs and so truth of formulas is also evaluated at ordered pairs in structures. The accessibility relations of the modalities will be restricted by keeping one of the coordinates of the two-dimensions constant.

Such logics may be of interest to those investigating natural language semantics, describing changes in temporal information contained in databases, using interval temporal logics to describe the relationships between processes or states of extended duration, combining temporal logic with logics of possibility, knowledge or belief, describing systems of parallel processes or trying to find modal approximates to the first-order logic of two variable symbols.

In this paper we will briefly look at axiomatizations for these logics using

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