

Simple groups definable in O -minimal structures

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1 Introduction

In this survey article we wish to describe our classification, or identification, of those (definably) simple groups G which are definable in O -minimal structures. Our basic result says that any such group is a linear semialgebraic group over some real closed field R . An even sharper result says that the structure (G, \cdot) is *bi-interpretable* with either the field R or with the field $R(i)$, where i is the square root of -1 . These results will appear in [8] and [9]. (It should be stated here that for technical reasons our hypothesis on the group G is that it is actually definable in M rather than interpretable. We, however, expect all the results to go through under only the interpretability hypothesis.)

Our results yield an O -minimal analogue of the well-known Cherlin conjecture. Recall that the (as yet unproved) Cherlin conjecture states any (noncommutative) simple group of finite Morley rank is a linear algebraic group over an algebraically closed field. It can be shown that if G is a simple group of finite Morley rank, then there is some strongly minimal set D such that G can be defined in D . Thus the finite Morley rank hypothesis in the Cherlin conjecture can be replaced by : G is definable in some strongly minimal structure. So it is quite natural to ask the O -minimal analogue: what are the simple groups definable in O -minimal structures? And this is what we have answered. In fact our second result (the bi-interpretability result mentioned above) clearly yields the full Cherlin conjecture for simple groups of finite Morley rank which happen to be definable in O -minimal structures. Any such group is a linear algebraic group over an algebraically closed field of characteristic 0. Moreover this yields even a model-theoretic *characterisation* of simple algebraic groups over algebraically closed fields of characteristic 0.

For the remainder of this introduction we shall define our terms and give

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