Free monoid completeness of the Lambek calculus allowing empty premises

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ABSTRACT We prove that the Lambek syntactic calculus allowing empty premises is complete with respect to the class of all free monoid models (i. e., the class of all string models, allowing the empty string).

Introduction

Lambek syntactic calculus (introduced in [7]) is one of the logical calculi used in the paradigm of categorial grammar for deriving reduction laws of syntactic types in natural and formal languages. The intended models for these calculi are free semigroup models (also called language models or string models), where each syntactic category is interpreted as a set of non-empty strings over some alphabet of symbols. Models for Lambek calculus were studied in [2], [3], [4], [5], [6], etc. Completeness of the Lambek calculus with respect to string models was proved in [9], [10], and [11]. Closely related is the result about completeness with respect to relational semantics [8].

There is a natural modification of the original Lambek calculus, which we call the Lambek calculus allowing empty premises (cf. [2, p. 44]). This calculus appears to be a fragment of the noncommutative linear logic. The natural class of string models for the Lambek calculus allowing empty premises is the class of all free monoid models, where also empty string is allowed.

In this paper we prove that the Lambek calculus allowing empty premises is complete with respect to these models.

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