Satisfaction classes and automorphisms of models of PA

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1 Introduction

In recent years we have learned a lot about countable recursively saturated models of PA and their automorphisms. We know that there are continuum many nonisomorphic automorphism groups of such models, and we know that each of them contains a copy of the automorphism group of the order preserving permutations of the rationals. We can classify the closed normal subgroups of a given automorphism group, and we have a great deal of information about open maximal subgroups. We know much about the model theory of the arithmetically saturated countable models of PA; in particular we know that the automorphism group of a countable arithmetically saturated model of PA has the small index property. But still many questions remain open: Can we classify all normal and all maximal subgroups of the automorphism group a given model? Do non arithmetically saturated models have the small index property? Is the automorphism group of a countable recursively saturated model decidable?

Many other results and questions could be mentioned here. However, the purpose of this paper is not to give a complete survey; this has been done recently by Kotlarski in [17]. Instead, I will concentrate on a specific feature of countable recursively saturated models of PA — inductive satisfaction classes and their use.

My goal is twofold. Often satisfaction classes allow one to give easy answers to questions that otherwise seem difficult. A list of examples is presented in section 4. I hope those who work in the model theory of recursively saturated models of PA will find this list useful. The other goal is to propose the following problem. Much of the model theory of countable recursively saturated models of PA is based on specific techniques (resplendency arguments, special 'back-and-forth' constructions), but a significant number of results can be obtained as corollaries of classical results concerning models of PA* applied to structures of the form (M, S), where M is a recursively

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