On “star” schemata of Kossak and Paris

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ABSTRACT Kossak and Paris introduced the “star” versions of the Induction and Collection schemata for Peano arithmetic, in which one admits, as extra parameters, subsets of a given nonstandard Peano model coded in a fixed elementary end extension of the model. We prove that the “star” schemata are not finitelly axiomatizable over recursively saturated models. A partial solution of a conjecture of Kossak and Paris is obtained.

Introduction

Kossak and Paris [2] have suggested the study of properties of second–order **PA** structures of the form $\langle M; N/M \rangle$, where M and N are nonstandard models of the Peano arithmetic, **PA**, N being an end extension of M (so that M is an initial segment of N), and N/M is the collection of all sets $X \subseteq M$ of the form $X = X' \cap M$, where $X' \subseteq N$ is an N-finite set (*i. e.* X' is coded in N as a finite set by some $a \in N$).

Let $\Sigma_n[N/M]$ denote the extension of the class of Σ_n formulas of the **PA** language by elements of M occuring in the usual way and sets $X \in N/M$ used as extra second–order parameters (with no quantification over them allowed).

This enrichment of the language leads us to the question: are the `Induction` and `Collection` schemata, restricted to the class of $\Sigma_{n+1}[N/M]$ formulas, really stronger than those restricted to $\Sigma_n[N/M]$ formulas ? Kossak and Paris obtained (see [2]) positive answers for the case when $n = 1$ or 2, and formulated it as a conjecture that the result should be true for all n.

This note is written to present a partial answer. We prove that, at least

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