Complete Sets and Structure in Subrecursive Classes

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ABSTRACT In this expository paper, we investigate the structure of complexity classes and the structure of complete sets therein. We give an overview of recent results on both set structure and class structure induced by various notions of reductions.

1 Introduction

After the demonstration of the completeness of several problems for **NP** by Cook [Coo71] and Levin [Lev73] and for many other problems by Karp [Kar72], the interest in completeness notions in complexity classes has tremendously increased. Virtually every form of reduction known in computability theory has found its way to complexity theory. This is usually done by imposing time and/or space bounds on the computational power of the device representing the reduction.

Early on, Ladner et al. [LLS75] categorized the then known types of reductions and made a comparison between these by constructing sets that are reducible to each other via one type of reduction and not reducible via the other. They however were interested just in the relative strength of the reductions and not in comparing the different degrees of complete sets that are induced by these reducibilities. This question was picked up much later by Watanabe [Wat87] for deterministic exponential time and others following him for other classes. A recent survey of this can be found in [Buh93]

Complete sets under some type of reduction form an interesting, rich, and hence much studied subject in complexity theory. A complete set can be viewed as representing an entire complexity class. Through the translation of the reduction one can with the help of the complete set, decide all questions of membership for any set in the class. The resource bound on the reduction is (to be of interest) always much less than the resource

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