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Properly Embedded Minimal Planar Domains with Infinite Topology are Riemann Minimal Examples

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Abstract. These notes outline recent developments in classical minimal surface theory that are essential in classifying the properly embedded minimal planar domains $M \subset \mathbb{R}^3$ with infinite topology (equivalently, with an infinite number of ends). This final classification result by Meeks, Pérez, and Ros [**64**] states that such an M must be congruent to a homothetic scaling of one of the classical examples found by Riemann [**87**] in 1860. These examples $\mathcal{R}_s, 0 < s < \infty$, are defined in terms of the Weierstrass $\mathcal{P}$-functions $\mathcal{P}_t$ on the rectangular elliptic curve $\frac{\mathbb{C}}{\langle 1, t\sqrt{-1}\rangle}$, are singly-periodic and intersect each horizontal plane in $\mathbb{R}^3$ in a circle or a line parallel to the x-axis. Earlier work by Collin [**22**], López and Ros [**49**] and Meeks and Rosenberg [**71**] demonstrate that the plane, the catenoid and the helicoid are the only properly embedded minimal surfaces of genus zero with finite topology (equivalently, with a finite number of ends). Since the surfaces $\mathcal{R}_s$ converge to a catenoid as $s \to 0$ and to a helicoid as $s \to \infty$, then the moduli space $\mathcal{M}$ of all properly embedded, non-planar, minimal planar domains in $\mathbb{R}^3$ is homeomorphic to the closed unit interval $[0, 1]$.

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Contents

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