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Notes on the Seiberg-Witten Equations, the Weinstein Conjecture and Embedded Contact Homology

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These notes are supplementary to the author's lectures at the 2007 conference on Current Developments in Mathematics, held on November 16, 17 in Cambridge Massachusetts. They outline various connections between the three subjects that are listed in the title.

My purpose in writing these notes is to give an indication of how the Seiberg-Witten equations on a 3-dimensional manifold can see something of the geometry of a contact 1-form. The new ideas for 3-dimensions led the author to a proof of the 3-dimensional version of the Weinstein conjecture [**W**]:

THEOREM. *The Reeb vector field for any given contact* 1*-form on any given compact* 3*-manifold has at least one closed, integral curve.*

The proof appears in [**T1**]; and [**T2**] says more about such integral curves.

Very much related to the story told here is the now decade old story explains how the Seiberg-Witten equations on a 4-dimensional symplectic manifold see something of the symplectic structure [**T3**], [**T4**]. The three and four dimensional stories are connected via a conjectured isomorphism between the Seiberg-Witten Floer cohomology and Michael Hutching's embedded contact homology [**H**], [**HS**], [**HT**]. I say something at the end about embedded contact homology.

A great deal was known about the Weinstein conjecture in dimension 3 prior to [**T1**]; much of this the pioneering work of Helmut Hofer. Hofer proved the conjecture for a huge class of contact structures; in particular contact structures with over-twisted 2-plane fields [**Hof1**]. He also proved the conjecture for any contact structure on S^3 and for any 3-manifold with non-zero π_2. Other recent work on this conjecture includes [**ACH**], [**Ch**], [**CH**], [**Ga**], and [**Hon**]. Hofer's ICM article [**Hof2**] and his reviews [**Hof3**], [**Hof4**]

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