

A biased view of symplectic cohomology

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1. Introduction

Symplectic cohomology is an invariant of a certain kind of symplectic manifolds (open, or with boundary). It is comparatively easy to define, being a variation on classical Hamiltonian Floer homology. Moreover, its behaviour reflects important aspects of symplectic topology in a fairly direct way. For instance, this applies to the fundamentally trivial nature of subcritical Stein manifolds, and to the importance of exact Lagrangian submanifolds, which are reflected in (nontrivial) vanishing resp. non-vanishing theorems for symplectic cohomology. In spite of this, and of the many successful early applications, the theory has not received the same level of attention as, say, Gromov-Witten theory or SFT (symplectic field theory).

These lecture notes are an attempt to advertise the breadth and attractiveness of symplectic cohomology, by stressing connections with various parts of symplectic topology and algebraic geometry. Because of this specific aim, our account may appear somewhat unbalanced (whence the title). On one hand, it includes a certain amount of previously unpublished material (various parts of this are due to Mark McLean, Ivan Smith, and the author; I have tried to mark clearly those places where I am borrowing other people's work). On the other hand, the exposition omits many technical details, and important classical results are stated entirely without proof. Fortunately, there are other surveys which perform much better in these respects, for instance [40, 53]. There is also some very recent work which could not be included in these notes, such as that of Bourgeois-Oancea relating symplectic cohomology with contact homology [5, 4]. Again, others will make up for this deficiency (Cieliebak and Oancea are preparing a paper which will explain this and other SFT-inspired work on symplectic cohomology).