

Gauge Theory, Ramification, And The Geometric Langlands Program

Sergei Gukov and Edward Witten

In the gauge theory approach to the geometric Langlands program, ramification can be described in terms of “surface operators,” which are supported on two-dimensional surfaces somewhat as Wilson or ’t Hooft operators are supported on curves. We describe the relevant surface operators in $\mathcal{N} = 4$ super Yang-Mills theory, and the parameters they depend on, and analyze how S -duality acts on these parameters. Then, after compactifying on a Riemann surface, we show that the hypothesis of S -duality for surface operators leads to a natural extension of the geometric Langlands program for the case of tame ramification. The construction involves an action of the affine Weyl group on the cohomology of the moduli space of Higgs bundles with ramification, and an action of the affine braid group on A -branes or B -branes on this space.

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