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The Sato-Tate conjecture

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0. Introduction

In April 2006, Richard Taylor has completed the proof of the Sato-Tate conjecture for elliptic curves over totally real fields, under a mild assumption (see below). This is the completion of a project started several years ago by Taylor and Michael Harris, aiming at developing the higher-dimensional automorphic deformation theory of Galois representation, with its applications to cases of Langlands functoriality – here, "Sato-Tate". I will report specifically on 3 papers:

Clozel, Harris, Taylor, *Automorphy for some ℓ-adic lifts of automorphic* mod ℓ *representations* ([**CHT**]);

Harris, Shepherd-Barron, Taylor, *Ihara's lemma and potential automorphy* ([**HSBT**]);

Taylor, *Automorphy for some ℓ-adic lifts of automorphic* mod ℓ *representations* II ([**T**]).

Together the three papers give a proof of "Sato-Tate" (with the restriction alluded to). The proof is based on the continuous development, since '95, of Wiles' deformation method. After the initial impetus by Wiles and Taylor-Wiles, I see two strands of development, characterized by:

• The systematic study, and use, of forms on **higher** groups and their associated Galois representations (Harris, Taylor).

• Deep technical progress (in degree 2, but this adapts to the higher, dimensional situation) in the Taylor-Wiles method: here fundamental contributions are due to Taylor, Diamond, Fujiwara, Skinner-Wiles, Kisin.

The purpose of these notes is to explain the result, even for a reader who is not familiar with the modern theory of automorphic forms and