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Modularity of 2-dimensional Galois representations

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Introduction

Our aim is to explain some recent results on modularity of 2-dimensional potentially Barsotti-Tate Galois representations. That such representations should arise from modular forms is a special case of a remarkable conjecture of Fontaine and Mazur [**FM**]. One of its concrete consequences is that if $A/\mathbb{Q}$ is an abelian variety of GL_2-type, then A is a subquotient of a product of Jacobians of modular curves.

The first breakthrough in the direction of this conjecture was the work of Wiles and Taylor-Wiles [**Wi**], [**TW**], which established that (under mild hypothesis) the conjecture holds for 2-dimensional p-adic representations ρ which are Barsotti-Tate at p provided that the associated mod p representation $\bar{\rho}$ is modular and irreducible. These results were extended by a number of authors [**Di 1**], [**CDT**], [**BCDT**], and a lifting theorem of this type for fairly general potentially Barsotti-Tate representations was proved in [**Ki 1**]. For ordinary representations with $\bar{\rho}$ reducible, the conjecture was proved by Skinner-Wiles [**SW**].

The condition that $\bar{\rho}$ was modular could be verified in certain special cases. The results mentioned in the previous paragraph were then sufficient to deduce the conjecture of Shimura-Taniyama-Weil that any elliptic curve over $\mathbb{Q}$ is modular. The case of semi-stable elliptic curves was established by Wiles [**Wi**] and the general case by Breuil-Conrad-Diamond-Taylor [**BCDT**].

However, Serre [**Se 1**] had conjectured that any two-dimensional mod p representation with odd determinant was modular. A few years ago Taylor established a weaker form of this conjecture [**Ta 1**], [**Ta 2**], which asserts that for some totally real field F in which p is unramified, $\bar{\rho}|_F$ arises from a Hilbert modular form. Combining this with the kind

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