IP Current Developments in Mathematics, 2004

Applications of minimal surfaces to the topology of three-manifolds

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1. Introduction

In this paper, I will mention some applications of minimal surfaces to the geometry and topology of three-manifolds that I discussed in my lecture at the Current Developments in Mathematics Conference for 2004.

The first important application of minimal surfaces to the geometry of three-manifolds was given by Schoen and Yau [**22**] in their study of Riemannian three-manifolds of positive scalar curvature and their related proof of the positive mass conjecture in general relativity. The techniques that they developed in their proof of this conjecture continue to be useful in studying relationships between stable minimal surfaces and the topology of Riemannian manifolds.

Around 1978, Meeks and Yau gave geometric versions of three classical theorems in three-dimensional topology. These classical theorems concern the existence of certain embedded surfaces. In the geometric versions of these theorems, Meeks and Yau proved the existence of essentially cononical solutions, which are given by area minimizing surfaces. They referred to these theorems as the Geometric Dehn's Lemma, Geometric Loop Theorem and the Geometric Sphere Theorem.

As an application of these special geometric minimal surface solutions to these classical topological theorems, Meeks and Yau gave new equivariant versions of these theorems in the presence of a differential finite group action. Their Equivariant Loop Theorem turned out to be the final missing step in the solution of the Smith Conjecture concerning the standardness of the smooth action of finite cyclic groups on the

This material is based upon work by the NSF under Award No. DMS - 0405836. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the NSF.