A PRIORI BOUNDS FOR THE RICCATI EQUATION

R. S. BUCY
UNIVERSITY OF SOUTHERN CALIFORNIA

1. Introduction and general results

Since the appearance of [6] and [7], the theory of linear filtering has experienced a renaissance. This theory, although evidently well known to statisticians in terms of "least squares estimates," has found many applications in the early sixties, largely because of the realization and synthesis methods provided in [6] and [7]. For an indication of some of the aerospace applications to guidance of spacecraft, the interested reader may find detailed information in [3]. Although the theory of linear filtering has changed little from that given in [7] for the continuous time problem, the practical realization of the so-called "correlated noise problem" as treated *mathematically* in [6], has recently found a solution in [4]. The full solution of this discrete time filtering problem and its meaning is described in detail in [5]. For readers desirous of a survey of recent results in linear and nonlinear filtering, it is available in [5], while more detailed information can be found in [3].

In this paper, our interest will center on the discrete matrix Riccati equation with emphasis on the study of the asymptotic behavior of its covariance matrix solution. A major tool in this study will be the Duffin parallel resistance of two nonnegative definite matrices A and B denoted by $A:B$. This operation is described in detail in [1] and provides for us a link between the Riccati equation and the classical continued fraction theory described in [9] and [10].

We have undertaken to study the discrete Riccati equation from the point of view of continued fractions because this technique provides considerable generality in that the nonsingular theory becomes a rather special case (see [3], Chapter 5) and much deeper results are obtained for singular problems; also the methods are striking generalizations of classical continued fraction methods.

We will be concerned with the cone C of $d \times d$ real entry symmetric nonnegative definite matrices. The cone C induces a natural partial ordering as for $A \in C$ and $B \in C$, $A \geqq B$ when and only when $A - B \in C$. The object of study will be the map of C;

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