LOCAL ASYMPTOTIC MINIMAX AND ADMISSIBILITY IN ESTIMATION

JAROSLAV HÁJEK
FLORIDA STATE UNIVERSITY
and
CHARLES UNIVERSITY, PRAGUE

1. Introduction

In their vigorous search for an adequate asymptotic theory of estimation, statisticians have tried almost all their methodological tools: prior densities, minimax, admissibility, large deviations, restricted classes of estimates (invariant, unbiased), contiguity, and so forth. The resulting body of knowledge is somewhat atomized and a certain synthetic work seems to be needed. In this section, let us try to single out several pieces of the jigsaw puzzle and combine them into a logically connected theory. Along with this we shall criticize some other approaches and make a few historical remarks.

Consider a fixed parametric space Θ and a sequence of experiments described by families of densities $p_n(x_n, \theta)$, say $p_n(x_n, \theta) = \prod_{i=1}^n f(y_i, \theta)$, where $x_n = (y_1, \cdots, y_n)$. First of all, it is necessary to single out "regular cases." This should not be done only formally, for example, only in terms of θ derivatives of $f(y, \theta)$. The statistical essence of regularity consists in the possibility of replacing the family of distributions by a normal family in a local asymptotic sense. Loosely speaking, given a point $t \in \Theta$ and a small vicinity V_t of t, the quantity

$$\Delta_{n,t} = n^{-1/2} \frac{\partial}{\partial \theta} \log p_n(x_n, \theta)\Big|_{\theta = t} \tag{1.1}$$

should be approximately sufficient and normal (with constant covariance and expectation linear in θ) for $\theta \in V_t$ and n large. See Section 3 for a precise definition. The idea of approximating a general family by a normal family was first formulated by A. Wald [19], and then sophisticatedly developed by L. LeCam [12], [13], [15]. In spite of its importance, the idea has not yet found its way into current textbooks.

The next step is to get rid of ill behaved estimates and to characterize optimum ones. This may be achieved by scrutinizing an arbitrary sequence of estimates T_n from the point of view of minimax and admissibility, again in a *local* asymptotic sense. Theorem 4.1 below entails that there is a lower bound for asymptotic local maximum risk and that this bound may be achieved only if

Research sponsored in part by National Science Foundation Grant # GU 2612 to the Florida State University and in part by the Army, Navy and Air Force under Office of Naval Research Contract Number NONR 988(08), Task Order NR 042-004.