TESTS FOR MONOTONE FAILURE RATE

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1. Introduction and summary

In applications such as life-testing and reliability, a useful characterization of distributions is in terms of their failure rates. The failure rate $q(t)$ of a distribution $F(t)$ having a density $f(t)$ is defined by $q(t) = f(t)/\{1 - F(t)\}$ for t such that $F(t) < 1$. The probabilistic interpretation is that corresponding to a failure distribution $F(t)$, $q(t)$ represents the conditional probability density of failure given that failure has not yet occurred by time t.

It is easy to verify that the exponential distribution with density

$$f(t) = \lambda e^{-\lambda t} \tag{1.1}$$

has a constant failure rate λ. Physically this might correspond to a situation in which the object fails if a sufficiently large environmental stress occurs, such stresses being distributed according to a Poisson process. It is assumed that the object develops no greater propensity toward failure as time elapses.

In many physical situations the object does become more vulnerable to failure with increasing age. This is characteristic of objects subject to wear-out—moving parts, human beings past youth, and so on. In such situations one would expect the failure distribution to be characterized by an increasing failure rate. Examples of such distributions are the gamma with density

$$f(t) = \frac{\lambda(\lambda t)^{\alpha-1}e^{-\lambda t}}{\Gamma(\alpha)}, \qquad \lambda > 0, \quad \alpha \geq 1, \quad t \geq 0, \tag{1.2}$$

and the Weibull with density

$$f(t) = \lambda\alpha t^{\alpha-1}e^{-\lambda t^{\alpha}}, \qquad \lambda > 0, \quad \alpha \geq 1, \quad t \geq 0. \tag{1.3}$$

In certain situations, however, it is reasonable to expect that the failure rate will decrease, at least over a certain interval of time. Thus, during the early months of human life, as a result of infancy diseases, the failure rate actually decreases. For certain electronic components, manufacturing defects tend to cause failure early in life, so that the failure rate may be higher during the initial period of usage. Materials which become work-hardened may exhibit a decreasing failure rate during a certain interval of time.

For both theoretical and practical reasons, it is important to distinguish between the case of monotone nondecreasing failure rate (referred to hereafter as