ANALYSIS ON HILBERT SPACE WITH REPRODUCING KERNEL ARISING FROM MULTIPLE WIENER INTEGRAL

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1. Introduction

The multiple Wiener integral with respect to an additive process with stationary independent increments plays a fundamental role in the study of the flow derived from that additive process and also in the study of nonlinear prediction theory. Many results on the multiple Wiener integral have been obtained by N. Wiener [14], [15], K. Itô [5], [6], and S. Kakutani [8] by various techniques. The main purpose of our paper is to give an approach to the study of the multiple Wiener integral using reproducing kernel Hilbert space theory.

We will be interested in stationary processes whose sample functions are elements in E^* which is the dual of some nuclear pre-Hilbert function space E. For such processes we introduce a definition of *stationary process* which is convenient for our discussions. This definition, given in detail by section 2, definition 2.1, is a triple $\mathbf{P} = (E^*, \mu, \{T_t\})$, where μ is a probability measure on E^* and $\{T_t\}$ is a flow on the measure space (E^*, μ) derived from shift transformations which shift the arguments of the functions of E.

In order to facilitate the discussion of the Hilbert space $L_2 = L_2(E^*, \mu)$, we shall introduce a transformation τ defined by the following formula:

$$(\tau\varphi)(\xi) = \int_{E^*} e^{i\langle x,\xi\rangle}\varphi(x)\mu(dx) \qquad \text{for} \quad \varphi \in L_2, \tag{1.1}$$

where $\langle\cdot, \cdot\rangle$ denotes the bilinear form of $x \in E^*$ and $\xi \in E$. This transformation τ from L_2 to the space of functionals on E is analogous to the ordinary Fourier transform. By formula (1.1) and a requirement that τ should be a unitary transformation, $\mathfrak{F} \equiv \tau(L_2(E^*, \mu))$ has to be a Hilbert space with reproducing kernel $C(\xi - \eta)$, $(\xi, \eta) \in E \times E$, where C is the *characteristic functional* of the measure μ defined by

$$C(\xi) = \int_{E^*} e^{i\langle x,\xi\rangle}\mu(dx), \qquad \xi \in E. \tag{1.2}$$