ON A CLASS OF TWO-SAMPLE
BIVARIATE NONPARAMETRIC TESTS

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1. Introduction

The object of the present investigation is to propose and study a general class of nonparametric tests for the various types of problems that may usually arise in the case of two independent samples with bivariate observations. For this purpose, the concept of permutation tests has been used in the formulation of a class of tests based on appropriate generalized U-statistics, and the theory of permutation distribution of such generalized U-statistics has been developed further.

The advent of the theory of nonparametric methods in multivariate analyses may be regarded to be still in a more or less rudimentary stage, and only a few nonparametric contenders of some standard parametric multivariate procedures are available in the literature. The up-to-date development of distribution-free techniques in this field of research relates specifically to the problem of location in the single, as well as multisample case, and the problem of independence in the single sample case. In this study, I have confined myself to the multisample case only.

The earliest work on this line is the permutation test based on Hotelling's T^2-statistic, proposed and studied by Wald and Wolfowitz [25], in as early as 1944. This test is, however, a strictly value-permutation test and is subject to the usual limitations of this type of tests. Following this, there is a gap of nearly twenty years, during which practically no nonparametric test has evolved in this field. However, very recently, some attention has been paid to the development of nonparametric methods in multivariate multisample analyses.

Some genuine distribution-free tests for location in the bivariate two-sample, as well as p-variate c-sample $(p, c \geq 2)$, case have been proposed and studied by Chatterjee and Sen [2], [4]. On the other hand, some tests for the same problem, which are only asymptotically distribution-free, have been considered by Bhapkar [1]. Chatterjee and Sen [3] have also considered some exact distribution-free tests for the two-sample bivariate association problem, and some of these tests have been extended to the c-sample case by Sen [22].

However, all these tests are based on specific forms of test criteria and relate specifically to the problem of location and association. No attempt has yet been