APPROXIMATIONS IN INFORMATION THEORY

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1. Introduction

In the papers [8]–[11], [14] the author studied stochastic processes and channels, stationary, or nonstationary, with discrete time and arbitrary sets of states. In these papers, for regular processes and channels, two basic theorems of Shannon type [15] are proved for the case in which the states of the process and the channel input states are discrete and the output states arbitrary.

In this study, the essential role of the differential entropy of probability fields, processes, and channels appears. Obviously, if the sets of states are discrete, instead of the differential entropy, the correspondent entropy appears. Here we study the problem of approximation of processes with continuous sets of states by discrete processes and also of channels with continuous input-sets by channels with discrete input-sets.

In this study an essential role is played by the concept of ϵ-entropy of a set, of a probability field, of a channel, and of a complex source-channel. We may observe also the role played by differential entropy in the approximation problem. The constructions used here in the approximation problems are such that the essential properties of the given object are preserved.

2. The differential entropy of probability fields

Let us consider the measure space (X, S, μ) where X is a set of elements x and S a σ-algebra of subsets of X and μ a measure in S. Over X let us consider a probability field A, defined by the probability density $p(x)$ with respect to μ. By M we denote the expectation.

DEFINITION 2.1. *The value* $h(A) = -M \log p(x)$ *is the differential entropy of* A *with respect to* μ.

Obviously, $h(A)$ exists only if $M |\log p(x)| < +\infty$, and from $|h(A)| < M |\log p(x)|$ it follows that in this case it is finite.

Let (X, S, μ), (Y, Σ, ν) be measure spaces, $\pi(x, y)$ the probability density of some field C over their product, A the field defined by the probability density $p(x)$ induced by $\pi(x, y)$ in X, and $q_x(y) = \pi(x, y)/p(x)$ the conditional probability density of some probability field B_x over Y. We denote $C = AB$ (the union).