ON TESTING FOR NORMALITY

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1. Introduction

The majority of the goodness-of-fit problems arising in practice involves nuisance parameters. On the other hand, the majority of the results which have appeared within this field deal with simple hypothesis testing only. Among the relatively few results concerning the first-mentioned, more general problem, the most known and applied one is the modification given by R. A. Fisher to Karl Pearson's χ^2-test (see for example, [4], pp. 424–434). As it is well known, this modification consists of replacing the unknown parameters by their estimates; the distribution of the modified test statistic (at least its approximate distribution) was determined. The same way was followed by Kac, Kiefer, and Wolfowitz [7] and Darling [5] concerning the Cramér-von Mises test and the Kolmogorov test. Computational difficulties, however, prevented them from providing tables having sufficient range and accuracy for practical purposes.

But even if these difficulties could be overcome in the future, neither they nor Fisher's method work in some nonsimple sample cases.

Considering this fact as well as the disadvantages of the χ^2-test (see [7], pp. 191–192), other solutions of problems of this type seem to be of particular interest. The straightforward way is to find an equivalent simple hypothesis.

The basic theory and the most important results of this approach are dealt with in another paper appearing in this volume [12].

Solutions of this type form the subject of the present paper, but it is confined in a rather special direction. This specialization may be characterized by the aim of avoiding theoretical and computational difficulties and of utilizing the known results of the theory of goodness-of-fit tests as much as possible.

Therefore, we are interested in such equivalent (substitute) hypotheses which are of the form of goodness-of-fit problems. In other words, we want to provide a set of random variables which are distribution-free and independently and identically distributed in the case of the null hypothesis.

As a further specialization, we require that each of these transformed values should represent, in the same way, one of the original sample elements. The purpose of this restriction is that the test statistic made with the transformed variables should approximate the correct test one could form with the knowledge of the unknown constants. Therefore, the properties of the combined test, consisting of the transformation and the testing of the simple hypothesis, will be