LATTICE METHODS AND SUBMARKOVIAN PROCESSES

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1. Introduction

1.1. *Summary.* This paper is a contribution to the study by lattice methods of stationary submarkovian processes on a denumerable state space. Our restriction to such simple spaces is only for clarity; it is by no means a restriction on the validity of the lattice methods. We have tried to present in this paper a synthesis of results recently obtained in the field of submarkovian processes as well as a certain number of new results.

We thought it preferable to introduce vector lattices only through their cones of positive elements (to be called L-cones); the basic notions concerning them are briefly summed up in this introduction. No representation using Stonian spaces has been introduced in this work.

Let $P = \{P_t, t \geqq 0\}$ be a submarkovian process on the denumerable set A. Then to every stochastic function $\{X_t\}$ defined for $t > 0$, with transition laws given by P, is associated a family $f = \{f_t, t > 0\}$ of positive measures on A of total mass $\leqq 1$ (the instantaneous laws of the X_t) such that

$$f_s P_t = f_{s+t}, \qquad s, t > 0; \tag{1.1.1}$$

and conversely. There exists an initial distribution f_0 such that $f_t = f_0 P_t$ when, but only when, $\{X_t\}$ can also be defined for $t = 0$. Recent advances in the theory have shown the interest of considering solutions of (1.1.1) with no restriction on the total mass of the f_t (which may even be infinite); for these more general solutions of (1.1.1), we prove in section 2.1 the following three basic properties:

(a) the t-functions $f_t(i)$ are continuous on $[0, \infty)$;

(b) the set $F(P)$ of all solutions f is an L-cone;

(c) the Laplace transform maps a "bounded solution" f onto a solution $\hat{f} = \{\hat{f}_x, x > 0\}$ of the equations $\hat{f}_y[I + (y - x)R_x] = \hat{f}_x$, where $x, y > 0$, and conversely.

Since the positivity and the continuity properties of the matrices P_t are the essential conditions under which these statements are true, it is remarked in section 2.2 that similar statements are valid for the positive solutions $g = \{g_t, t > 0\}$ of $P_t g_s = g_{s+t}$ with $s, t > 0$.

Given two processes P (on A) and P' (on A'), families $C = \{C_{s,t}; s, t > 0\}$ of positive matrices on $A \times A'$ that satisfy the relations