ASYMPTOTIC EFFICIENCY AND LIMITING INFORMATION

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1. Introduction

In a recent paper [15], the author gave new formulations of the concepts of asymptotic efficiency and consistency, which seem to throw some light on the principle of maximum likelihood (m.l.) in estimation. An attempt was also made in that paper to link up the concept of asymptotic efficiency with the limiting information per observation contained in a statistic, as the sample size tends to infinity, information on an unknown parameter θ being defined in the sense of Fisher [5], [6].

The object of the present paper is to pursue the investigation of the earlier paper and establish some further propositions which might be of use in understanding the m.l. method of estimation.

The first proposition is concerned with the conditions under which i_T, the information per observation in a statistic T_n tends to i, the information in a single observation, as the sample size $n \to \infty$. It may be noted that i_T cannot exceed i for any n. A sufficient condition for convergence of i_T to i is

$$\left| n^{-1/2} \left(d \frac{\log L}{d\theta} \right) - \alpha - \beta n^{1/2}(T_n - \theta) \right| \to 0 \tag{1.1}$$

in probability, where L denotes the likelihood of θ given the sample and α, β are constants possibly depending on θ. This proposition was first proved by Doob [3], but his proof appears to be somewhat complicated. Under the same conditions assumed by Doob, the problem has been formulated in a more general way and a simple proof has been provided. It is observed that asymptotic efficiency of an estimator T_n may be defined as the property (1.1), or a less restrictive condition such as the asymptotic correlation between $n^{-1/2}(d \log L/d\theta)$ and $n^{1/2}(T_n - \theta)$ being unity, which imply that $i_T \to i$. This new definition of asymptotic efficiency is applicable to a wider class of statistics, while for the application of the usual definition in terms of the asymptotic variance of the estimator some regularity conditions on the estimator have to be imposed.

It is known that under some regularity conditions the m.l. estimate or an estimate obtained as a particular (consistent) root of the equation $(d \log L/d\theta) = 0$, has the property (1.1). But the m.l. estimate is only one member of a large class of estimates satisfying this property. The minimum chi-square, modified