WHY "SHOULD" STATISTICIANS AND BUSINESSMEN MAXIMIZE "MORAL EXPECTATION"?

J. MARSCHAK
COWLES COMMISSION

1. Introduction

1.1. The word "should" in the title of this paper has the same meaning as in the following sentences: "In building a house, why should one act on the assumption that the floor area of a room is the product and not the sum of its length and width?"; "If all A are B and all B are C, why should one avoid acting as if all C were A?" People may often act contrary to these precepts or norms but then we say that they do not act reasonably. To discuss a set of norms of reasonable behavior (or possibly two or more such sets, each set being consistent internally but possibly inconsistent with other sets) is a problem in logic, not in psychology. It is a normative, not a descriptive, problem.

1.2. The phrase "moral expectation" stems from the early students of probability who applied probabilities in their study of reasonable behavior of players in games of chance. Let the "prospect" P, that is, the probability distribution $P(X)$ of a random "outcome" X, depend upon a man's decision ("strategy") S:

$$P = P(X) = P(X; S). \tag{1.2:1}$$

Let the set $\mathcal{X}$ of all possible outcomes X be completely ordered by a relation $\mathfrak{g}$ ("read: as good as or better than"). Define a scalar function $u(X)$ on the set $\mathcal{X}$ as follows: for any pair, X_1 and X_2, in $\mathcal{X}$,

$$u(X_1) \geqq u(X_2) \quad \text{if } X_1 \mathfrak{g} X_2. \tag{1.2:2}$$

Then $u(X)$ is called the utility of X. It is a random variable whose distribution depends on the distribution P and hence on the strategy S. Its expected value,

$$Eu(X) \mid P(X; S) = \mu_u(S), \quad \text{say}, \tag{1.2:3}$$

is called the moral expectation of X. Define a space $\mathcal{S}$ whose elements S represent possible strategies. The title of the paper asks whether it is reasonable always to choose as one's strategy an element S^* of $\mathcal{S}$ whenever

$$\mu_u(S^*) > \mu_u(S') \tag{1.2:4}$$

where S' is any element of $\mathcal{S}$ distinct from S^*.

1.3. The "precept," always (that is, for any space $\mathcal{S}$) to maximize moral expectation, leads to inconsistent results unless all the utility functions considered

This article will be reprinted as Cowles Commission Paper, New Series, No. 53.