GALTON-WATSON PROCESSES WITH GENERATION DEPENDENCE

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1. Introduction

A Galton-Watson process Z_n can be thought of in the following way. There is one cell alive in generation zero. This cell dies and gives birth to a random number Z_1 of baby cells in the first generation. Each of these cells dies and gives birth to a random number of cells in the second generation. The number of cells in the second generation is Z_2. The process continues; Z_n is the number of cells in the nth generation. The number of daughters born to a cell is allowed to be a random variable whose distribution depends upon the generation of the cell in question. In this paper the following questions are answered under certain conditions.

(i) What are the mean and variance of Z_n?

(ii) Does $Z_n/E(Z_n)$ converge to a nonzero and nonconstant random variable W?

(iii) If the answer to (ii) is yes, what are the mean and variance of W?

(iv) What is the behavior of $P(Z_n \neq 0)$ for large n?

If X and Y are random variables and A and B denote events, then $E(X)$ is mean of X, Var (X) is the variance of X, $E(X|Y)$ is the conditional mean of X given Y, $P(A)$ is the probability that A happens, and $P(A|B)$ is the conditional probability that A happens, given that B occurs. This paper is the first chapter of [1].

2. Definition of Z_n, the probability generating function of Z_n, and the Markov nature of Z_n

First, Z_n is defined inductively. Let $X_{n,k}$, for $n = 0, 1, 2, \cdots, k = 1, 2, \cdots$, be a family of independent nonnegative integer valued random variables such that, for n fixed, $X_{n,k}$, $k = 1, 2, \cdots$, are identically distributed. Define $Z_0 = 1$, and having defined Z_n, define

$$(1) \qquad Z_{n+1} = \begin{cases} \sum_{k=1}^{Z_n} X_{n,k} & \text{if } Z_n \geqq 1, \\ 0 & \text{if } Z_n = 0. \end{cases}$$