A TIME DEPENDENT SIMPLE STOCHASTIC EPIDEMIC

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1. Introduction

Since the pioneer work of A. M. McKendrick in 1926, many authors have contributed to the advancement of the stochastic theory of epidemics, including Bartlett [4], Bailey [1], D. G. Kendall [12], Neyman and Scott [13], Whittle [16], to name a few. Mathematical complexity involved in some of the epidemic models has aroused the interest of many others. For example, the general stochastic epidemic model where a population consists of susceptibles, infectives, and immunes (see [2], p. 39), has motivated Kendall to suggest an ingenious device. Other authors also have investigated various aspects of the problem. (See, for example, Daniels [8], Downton [9], Gani [11] and Siskind [15].) The model discussed in the present paper deals with a closed population without removal of infectives, a special case of which has been studied very extensively by Bailey [3]. Following Bailey, we label it "a time dependent simple stochastic epidemic."

In a simple stochastic epidemic model, a population consists of two groups of individuals: susceptibles and infectives; there are no removals, no deaths, no immunes, and no recoveries from infection. At the initial time $t = 0$, there are N susceptibles and 1 infective. For each time t, for $t > 0$, there are a number of infectives denoted by $Y(t)$ and a number of uninfected susceptibles $X(t)$, with $Y(t) + X(t) = N + 1$, the total population size remaining unchanged. Our primary purpose is to derive an explicit solution for the probability distribution of the random variable $Y(t)$,

$$P_{1n}(0, t) = Pr\{Y(t) = n | Y(0) = 1\}, \qquad n = 1, \cdots, N + 1. \tag{1}$$

For each interval (τ, t), $0 \leqq \tau \leqq t < \infty$, and for each n, we assume the existence of a nonnegative continuous function $\beta_n(\tau)$ such that

$$\frac{\partial}{\partial t} P_{n,m}(\tau, t)\bigg|_{t=\tau} = \begin{cases} -\beta_n(\tau) & \text{for } m = n, \\ \beta_n(\tau) & \text{for } m = n + 1, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Under the assumption of homogeneous mixing of the population, we let