EQUILIBRIA FOR GENETIC SYSTEMS WITH WEAK INTERACTION

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1. Introduction

The following principle (stated here in rough form) bears many applications in the study of ecological and genetic systems.

PRINCIPLE I. *If a system of transformations acting on a certain set (in finite dimensional space) has a "stable" fixed point, then a slight perturbation of the system maintains a stable fixed point nearby.*

The theme of this principle is quite intuitive although care in its application and interpretation is vital. Its validity does not require the stability hypothesis to apply in a geometric sense. In fact, for numerous important nonrandom mating genetic models the stability of the relevant equilibrium is manifested only in an algebraic sense. The result is basic in the domain of global analysis and occurs in many other mathematical contexts as well. With the aid of this principle, we are able to establish the existence of equilibria for quite complicated genetic models and these have interesting interpretations for population phenomena.

A converse proposition to Principle I of considerable value in ascertaining *all* possible equilibria is also now stated in rough form. For a precise mathematical statement the reader should consult Karlin and McGregor [9].

PRINCIPLE II. *If $f(x)$ is a differentiable transformation acting on a certain set S (in finite dimensional space) having a finite number of fixed points, say $y_1, y_2, \cdots, y_r$, with the property that the linear approximation to $f(x)$ in the neighborhood of each fixed point has no eigenvalue of absolute value one, then a slight differentiable perturbation of $f(x)$ maintains* at most *a single fixed point $z_i \in S$ in the neighborhood of each y_i. Moreover, z_i is locally stable if and only if y_i is locally stable.*

It is worth noting that some fixed points of $f(x)$ (but none of the stable ones) may disappear under small perturbations.

We illustrate the scope of these principles by indicating the application to the investigation of three types of population models subject to a variety of genetic

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