CRITICAL AGE DEPENDENT BRANCHING PROCESSES

HOWARD J. WEINER
UNIVERSITY OF CALIFORNIA, DAVIS

1. Introduction

This paper is a survey of some recent work which generalizes standard results in the Bellman-Harris single type critical age dependent branching process, especially the asymptotic probability of nonextinction of the process, and a limiting conditional exponential limit law. Also included are new results combining existing extensions and suggestions for further research and techniques in relaxing conditions on the processes.

2. Definition

The classical Bellman-Harris age dependent branching process ([8], Chapter 6) is defined as follows. At time 0, one new born cell starts the process with nonlattice lifetime distribution function $G(t)$, with $G(0) = 0$. At the end of its life, the cell disappears and is replaced by k daughter cells with probability p_k, $k = 0, 1, 2, 3, \cdots$. Each daughter cell behaves independent of all other cells, and has the lifetime distribution $G(t)$. Denote by $h(s)$ the generating function

$$h(s) = \sum_{k=0}^{\infty} p_k s^k. \tag{2.1}$$

If $h'(1) \equiv m$, the mean number of daughter cells born to a parent cell, then the cases $m > 1$, $m = 1$, and $m < 1$ form a trichotomy for the behavior of the process in crucial respects, where $m = 1$ is the critical case (see [8]). We will consider now results for $Z(t)$, the number of cells alive at t.

3. Early results

When $m = 1$, $h^{(2)}(1) > 0$ and $h^{(3)}(1) < \infty$, and $G(t) = 1 - \exp\{-\lambda t\}$, Sevast'janov (see [8], Chapter 5) showed, by consideration of a differential equation satisfied by the generating function $F(s, t) \equiv \sum_{k=0}^{\infty} P[Z(t) = k]s^k$, that

$$\lim_{t\to\infty} tP[Z(t) > 0] = 2[\lambda h^{(2)}(1)]^{-1} \tag{3.1}$$

and

$$\lim_{t\to\infty} P[2(\lambda h^{(2)}(1)t)^{-1}Z(t) > u | Z(t) > 0] = \exp\{-u\}, \tag{3.2}$$

for $u \geqq 0$.