A CLASS OF STOPPING RULES FOR TESTING PARAMETRIC HYPOTHESES

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Let $f_\theta(x)$, $\theta \in \Omega$, be a one parameter family of probability densities with respect to some σ-finite measure μ on the Borel sets of the line. Denote by P_θ the probability measure under which random variables $x_1, x_2, \cdots$ are independent with the common probability density $f_\theta(x)$. Let θ_0 be an arbitrary fixed element of Ω and ε any constant between 0 and 1. We are interested in finding stopping rules N for the sequence $x_1, x_2, \cdots$ such that

$$P_\theta(N < \infty) \leqq \varepsilon \qquad \text{for every } \theta \leqq \theta_0, \tag{1}$$

and

$$P_\theta(N < \infty) = 1 \qquad \text{for every } \theta > \theta_0. \tag{2}$$

Among such rules, we wish to find those which in some sense minimize $E_\theta(N)$ for all $\theta > \theta_0$.

A method of constructing rules which satisfy (1) and (2) by using mixtures of likelihood ratios was given in [3]. Here we sketch an alternative method.

Let $\theta_{n+1} = \theta_{n+1}(x_1, \cdots, x_n)$ for $n = 0, 1, 2, \cdots$, be any sequence of Borel measurable functions of the indicated variables such that

$$\theta_{n+1} \geqq \theta_0. \tag{3}$$

In particular, θ_1 is some constant $\geqq \theta_0$. Define

$$z_n = \prod_1^n \frac{f_{\theta_i}(x_i)}{f_{\theta_0}(x_i)}, \qquad n = 1, 2, \cdots, \tag{4}$$

and for any constant $b > 0$, let

$$N = \begin{cases} \text{first } n \geqq 1 \text{ such that } z_n \geqq b, \\ \infty \text{ if no such } n \text{ occurs.} \end{cases} \tag{5}$$

We shall show that under a certain very general assumption on the structure of the family $f_\theta(x)$, the inequality (1) holds at least for all $b \geqq 1/\varepsilon$.

ASSUMPTION. *For every triple* $\alpha \leqq \gamma \leqq \beta$ *in* Ω,

$$\int \frac{f_\alpha(x) f_\beta(x)}{f_\gamma(x)}\, d\mu(x) \leqq 1. \tag{6}$$

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